Sonora High School Math Department
Precalculus
Summer Packet

The problems in this packet are designed to help you review topics from Algebra 2 that are important for your success in Precalculus. All work must be shown. You can write directly in the packet. Examples are provided to help assist you in remembering these topics. In addition, the websites listed below may also help you in remembering the topics.

http://www.cliffsnotes.com/study-guides/algebra/algebra-ii
http://tutorial.math.lamar.edu/Classes/Alg/Alg.aspx
https://www.khanacademy.org
http://patrickjmt.com
http://www.freemathhelp.com
http://www.coolmath.com
http://www.onlinemathlearning.com

Topic 1: The Distance Formula and Midpoint Formula

The Distance Formula

The distance d between the points (x_1, y_1) and (x_2, y_2) is as follows:

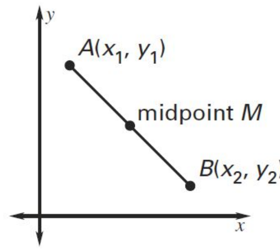
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The Midpoint Formula

The midpoint of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is as follows:

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Each coordinate of M is the mean of the corresponding coordinates of A and B.



Worked out examples:

Find the distance between $(4, -3)$ and $(6, 2)$.

SOLUTION

Let $(x_1, y_1) = (4, -3)$ and $(x_2, y_2) = (6, 2)$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Use distance formula.
$= \sqrt{(6 - 4)^2 + (2 - (-3))^2}$	Substitute.
$= \sqrt{2^2 + 5^2}$	Simplify.
$= \sqrt{4 + 25}$	Simplify.
$= \sqrt{29} \approx 5.39$	Use a calculator.

Find the midpoint of a line segment from $P_1 = (-5, 5)$ to $P_2 = (3, 1)$. Plot the points P_1 and P_2 and their midpoint. Check your answer.

We apply the midpoint formula (2) using $x_1 = -5$, $y_1 = 5$, $x_2 = 3$, and $y_2 = 1$. Then the coordinates (x, y) of the midpoint M are

$$x = \frac{x_1 + x_2}{2} = \frac{-5 + 3}{2} = -1 \quad \text{and} \quad y = \frac{y_1 + y_2}{2} = \frac{5 + 1}{2} = 3$$

That is, $M = (-1, 3)$.

Practice Problems

Find the distance and midpoint between the given points.

1. $(-1, 0)$ and $(2, 4)$
2. $(-4, -3)$ and $(6, 2)$
3. $(1.2, 2.3)$ and $(-0.3, 1.1)$
4. (a, a) and $(0, 0)$
5. Find all points having an x-coordinate of 9 whose distance from the point $(3, -2)$ is 10.

Topic 2: Intercepts

Find Intercepts from an Equation

The intercepts of a graph can be found from its equation by using the fact that points on the x-axis have y-coordinates equal to 0 and points on the y-axis have x-coordinates equal to 0.

Procedure for Finding Intercepts

1. To find the x-intercept(s), if any, of the graph of an equation, let $y = 0$ in the equation and solve for x.
2. To find the y-intercept(s), if any, of the graph of an equation, let $x = 0$ in the equation and solve for y.

Because the x-intercepts of the graph of an equation are those x-values for which $y = 0$, they are also called the **zeros** (or **roots**) of the equation.

Worked out Examples:

Find the x-intercept(s) and the y-intercept(s) of the graph of $y = x^2 - 4$.

To find the x-intercept(s), we let $y = 0$ and obtain the equation

$$\begin{aligned} x^2 - 4 &= 0 \\ (x + 2)(x - 2) &= 0 \quad \text{Factor.} \\ x + 2 = 0 \quad \text{or} \quad x - 2 &= 0 \quad \text{Zero-Product Property} \\ x = -2 \quad \text{or} \quad x &= 2 \end{aligned}$$

The equation has two solutions, -2 and 2. The x-intercepts (or zeros) are -2 and 2.

To find the y-intercept(s), we let $x = 0$ in the equation.

$$\begin{aligned} y &= x^2 - 4 \\ &= 0^2 - 4 = -4 \end{aligned}$$

The y-intercept is -4. ◀

Practice Problems

Determine the intercepts of each equation.

1. $y = x - 6$
2. $y = x^2 - 9$
3. $5x + 2y = 10$
4. $4x^2 + y = 4$

Topic 3: Finding the Slope (Rate of Change)

Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ be two distinct points. If $x_1 \neq x_2$, the **slope** $\boldsymbol{m}$ of the nonvertical line L containing P and Q is defined by the formula

$$\boxed{m = \frac{y_2 - y_1}{x_2 - x_1}, \qquad x_1 \neq x_2} \qquad \textbf{(1)}$$

If $x_1 = x_2$, L is a **vertical line** and the slope m of L is **undefined** (since this results in division by 0).

Worked out examples:

Find the slope of the line passing through $(-1, 4)$ and $(-3, 4)$.

Solution

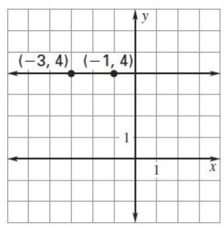
Let $(x_1, y_1) = (-1, 4)$ and $(x_2, y_2) = (-3, 4)$.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ Write formula for slope.

$= \dfrac{4 - 4}{-3 - (-1)}$ Substitute values.

$= \dfrac{0}{-2}$ Simplify.

Because the slope is zero, the line is horizontal.



Find the slope of the line passing through $(-3, 2)$ and $(1, 5)$.

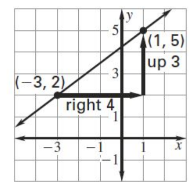
Solution

Let $(x_1, y_1) = (-3, 2)$ and $(x_2, y_2) = (1, 5)$.

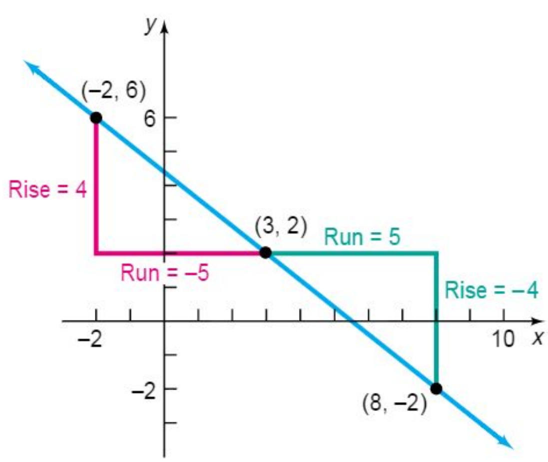
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ ← Rise: Difference of y-values ← Run: Difference of x-values

$= \dfrac{5 - 2}{1 - (-3)}$ Substitute values.

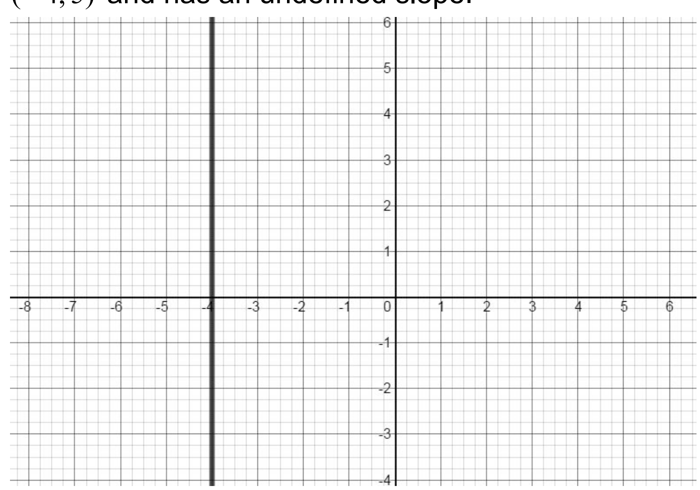
$= \dfrac{3}{1 + 3} = \dfrac{3}{4}$ Simplify. Slope is positive.



Draw a graph of the line that contains the point $(3, 2)$ and has a slope of $-\frac{4}{5}$.



Draw a graph of the line that contains the point $(-4, 5)$ and has an undefined slope.



Practice Problems

Find the slope of the line passing through the given pair of points.

1. $(-1, 1)$ and $(2, 3)$ 2. $(-4, 7)$ and $(5, 7)$ 3. $(4, 2)$ and $(3, 4)$ 4. $(2, 0)$ and $(0, 2)$

Draw a graph of the line that passes through the given point and has the given slope.

5. Slope 2 ; point $(-2, 3)$ 6. Slope $-\frac{1}{3}$; point $(4, 1)$

Topic 4: Graphing Lines in slope-intercept and standard form

Worked out examples:

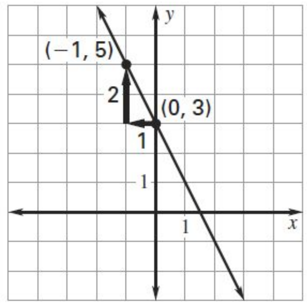
Graph $2x + y = 3$.

SOLUTION

1. Write the equation in slope-intercept form by subtracting $2x$ from each side.

   $y = -2x + 3$

2. The y-intercept is 3, so plot the point $(0, 3)$.
3. The slope is $-\frac{2}{1}$, so plot a second point by moving 1 unit to the left and 2 units up. This point is $(-1, 5)$.
4. Draw a line through the two points.



Graph $-2x + 3y = -6$.

SOLUTION

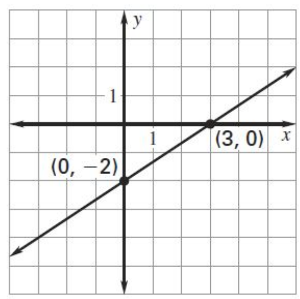
1. The equation is already in standard form.
2. $-2x + 3(0) = -6$ Let $y = 0$.

   $x = 3$ Solve for x.

   Plot $(3, 0)$, the x-intercept.
3. $-2(0) + 3y = -6$ Let $x = 0$.

   $y = -2$ Solve for y.

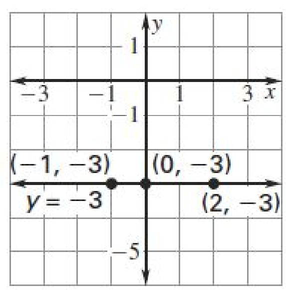
   Plot $(0, -2)$, the y-intercept.
4. Draw a line through the two points.



Graph the equation $y = -3$.

SOLUTION

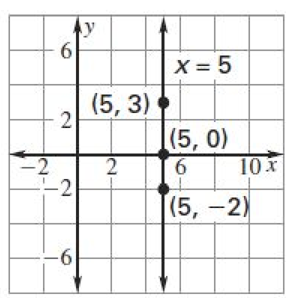
The y-value is always -3, regardless of the value of x. The points $(-1, -3)$, $(0, -3)$, $(2, -3)$ are some solutions of the equation. The graph of the equation is a horizontal line 3 units below the x-axis.



Graph the equation $x = 5$.

SOLUTION

The x-value is always 5, regardless of the value of y. The points $(5, -2)$, $(5, 0)$, $(5, 3)$ are some solutions of the equation. The graph of the equation is a vertical line 5 units to the right of the y-axis.



Practice Problems

Graph each line. Label the intercepts.

1. $y = 2x + 3$
2. $x = -3$
3. $3x - 2y = 6$
4. $y = 1$
5. $x + y = 0$
6. $x - \frac{2}{3}y = 4$
7. $-0.3x + 0.4y = 1.2$
8. $y = -\frac{2}{5}x - 1$

Topic 6: Writing Equations of Lines

Point–Slope Form of an Equation of a Line

An equation of a nonvertical line of slope m that contains the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \quad (2)$$

Slope–Intercept Form of an Equation of a Line

An equation of a line L with slope m and y-intercept b is

$$y = mx + b \quad (3)$$

Equation of a Vertical Line

A vertical line is given by an equation of the form

$$x = a$$

where a is the x-intercept.

Equation of a Horizontal Line

A horizontal line is given by an equation of the form

$$y = b$$

where b is the y-intercept.

Worked Out Examples:

Write an equation of the line that passes through $(-1, -3)$ and has a slope of 4. **Solution** $y - y_1 = m(x - x_1)$ Use point-slope form. $y + 3 = 4(x + 1)$ Substitute for m, x_1, and y. $y + 3 = 4x + 4$ Distributive property $y = 4x + 1$ Write in slope-intercept form.	Write an equation of the line that passes through $(-1, -3)$ and has a slope of 4. **Solution** $y - y_1 = m(x - x_1)$ Use point-slope form. $y + 3 = 4(x + 1)$ Substitute for m, x_1, and y. $y + 3 = 4x + 4$ Distributive property $y = 4x + 1$ Write in slope-intercept form.

Write an equation of the line that passes through $(-1, -3)$ and $(2, 4)$. **SOLUTION** First, find the slope by letting $(x_1, y_1) = (-1, -3)$ and $(x_2, y_2) = (2, 4)$. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-(-3)}{2-(-1)} = \frac{7}{3}$ Because you know the slope and a point on the line, use the point-slope form to find an equation of the line. $y - y_1 = m(x - x_1)$ Use point-slope form. $y + 1 = \frac{7}{3}(x + 3)$ Substitute for m, x_1, and y_1. $y + 1 = \frac{7}{3}x + 7$ Distributive property $y = \frac{7}{3}x + 6$ Simplify.	Find an equation of the horizontal line containing the point $(3, 2)$. Because all the y-values are equal on a horizontal line, the slope of a horizontal line is 0. To get an equation, we use the point–slope form with $m = 0$, $x_1 = 3$, and $y_1 = 2$. $y - y_1 = m(x - x_1)$ $y - 2 = 0 \cdot (x - 3)$ $m = 0, x_1 = 3$, and $y_1 = 2$ $y - 2 = 0$ $y = 2$

Practice Problems

Write an equation of the line with the given properties. Express your answer in slope-intercept form.

1. slope $= \frac{1}{2}$; containing the point $(3, -1)$
2. Containing the points $(-3, 4)$ and $(2, 5)$
3. slope $= -2$; $y-$intercept $= 4$
4. $x-$intercept $= -4$; $y-$intercept $= 4$
5. Slope undefined; containing the point $(3, 8)$

Topic 5: Parallel and Perpendicular Lines

Criterion for Parallel Lines

Two nonvertical lines are parallel if and only if their slopes are equal and they have different y-intercepts.

Criterion for Perpendicular Lines

Two nonvertical lines are perpendicular if and only if the product of their slopes is -1.

Worked Out Examples:

Write an equation of the line that passes through (3, 2) and is (**a**) perpendicular and (**b**) parallel to the line $y = -3x + 2$.

a. The given line has a slope of $m_1 = -3$. So, a line that is perpendicular to this line must have a slope of $m_2 = -\frac{1}{m_1} = \frac{1}{3}$. Because you know the slope and a point on the line, use the point-slope form with $(x_1, y_1) = (3, 2)$ to find an equation of the line.

$y - y_1 = m_2(x - x_1)$	Use point-slope form.
$y - 2 = \frac{1}{3}(x - 3)$	Substitute for m_2, x_1, and y_1.
$y - 2 = \frac{1}{3}x - 1$	Distributive property
$y = \frac{1}{3}x + 1$	Write in slope-intercept form.

b. For a parallel line use $m_2 = m_1 = -3$ and $(x_1, y_1) = (3, 2)$.

$y - y_1 = m_2(x - x_1)$	Use point-slope form.
$y - 2 = -3(x - 3)$	Substitute for m_2, x_1, and y_1.
$y - 2 = -3x + 9$	Distributive property
$y = -3x + 11$	Write in slope-intercept form.

Practice Problems

Write the equation of the line with the given properties in slope-intercept form.

1. Parallel to the line $x - 2y = -5$; containing the point $(3, -2)$
2. perpendicular to the line $y = 2x - 3$; containing the point $(-1, 2)$
3. Parallel to the line $y = 4$; containing the point $(2, 5)$
4. perpendicular to the line $x = 1$; containing the point $(4, -3)$

Topic 7: Writing Equations of Circles & Graphing Circles

The **standard form of an equation of a circle** with radius r and center (h, k) is

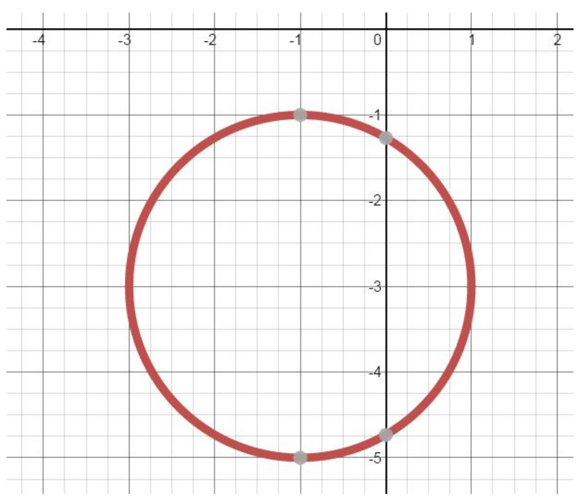
$$(x-h)^2 + (y-k)^2 = r^2 \qquad \textbf{(1)}$$

Worked Out Examples:

Write the equation $x^2 + y^2 + 2x + 6y + 6 = 0$ in standard form and graph.

To write the equation in standard form, complete the square for both the x- and y-terms:

$x^2 + y^2 + 2x + 6y + 6 = 0$	Write original equation.
$(x^2 + 2x) + (y^2 + 6y) = -6$	Group terms and subtract 6 from each side.
$(x^2 + 2x + 1) + (y^2 + 6y + 9) = -6 + 1 + 9$	Complete the square.
$(x + 1)^2 + (y + 3)^2 = 4$	Simplify.



Find the general equation of the circle whose center is $(1, -2)$ and whose graph contains the point $(4, -2)$.

To find the equation of a circle, we need to know its center and its radius. Here, we know that the center is $(1, -2)$. Since the point $(4, -2)$ is on the graph, the radius r will equal the distance from $(4, -2)$ to the center $(1, -2)$. See Figure 66. Thus,

$$\begin{aligned} r &= \sqrt{(4-1)^2 + [-2-(-2)]^2} \\ &= \sqrt{9} = 3 \end{aligned}$$

The standard form of the equation of the circle is

$$(x-1)^2 + (y+2)^2 = 9$$

Eliminating the parentheses and rearranging terms, we get the general equation

$$x^2 + y^2 - 2x + 4y - 4 = 0$$

Write the equation of the circle and graph.

1. $r = 4$; center $= (-5, -2)$

2. $r = \frac{1}{2}$; center $= (3, 0)$

Find the center and radius of each circle and then graph the circle.

3. $x^2 + (y-1)^2 = 1$

4. $x^2 + y^2 - 2x - 4y - 4 = 0$

5. $x^2 + y^2 - 6x + 2y + 9 = 0$

6. Find an equation of the circle with center $(1, 0)$ and containing the point $(-3, 2)$.